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[12 Nov 2020] • [AoE]

Performance Modeling of Streaming Kernels and Sparse Matrix-Vector Multiplication on A64FX

Christie Alappat, Jan Laukemann, Thomas Gruber, Georg Hager,
Gerhard Wellein, Nils Meyer, Tilo Wettig

NEWS

Japan Captures TOP500 Crown with Arm-Powered Supercomputer

June 22, 2020

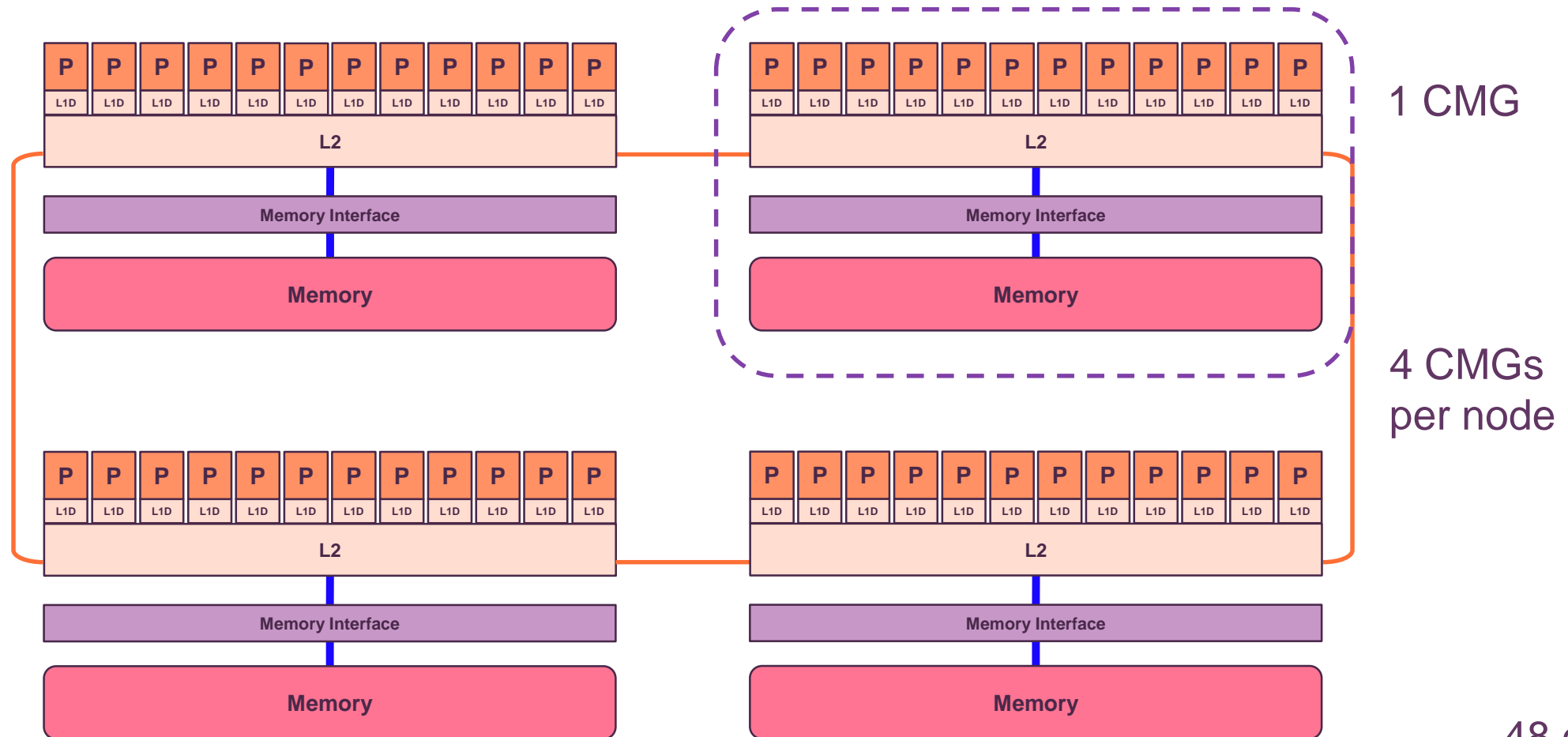
FRANKFURT, Germany; BERKELEY, Calif.; and KNOXVILLE, Tenn.—The 55th edition of the TOP500 saw some significant additions to the list, spearheaded by a new number one system from Japan. The latest rankings also reflect a steady growth in aggregate performance and power efficiency.

	SYSTEM	SPEED
1	Fugaku	Fujitsu A64FX (48C, 2.2GHz), Tofu Interconnect D
2	Summit	IBM POWER9 (20C, 3.0GHz), NVIDIA Volta GV50 (80C), Dual-Rail Mellanox EDR Infiniband
3	Sierra	IBM POWER9 (20C, 3.0GHz), NVIDIA Tesla V100 (80C), Dual-Rail Mellanox EDR Infiniband
4	Sunway TaihuLight	Shenwei SW26010 (280C, 1.45 GHz) Custom Interconnect
5	Tianhe-2A (Milkyway-2A)	Intel Ivy Bridge (20C, 2.2 GHz) & TH Express-2, Mellanox

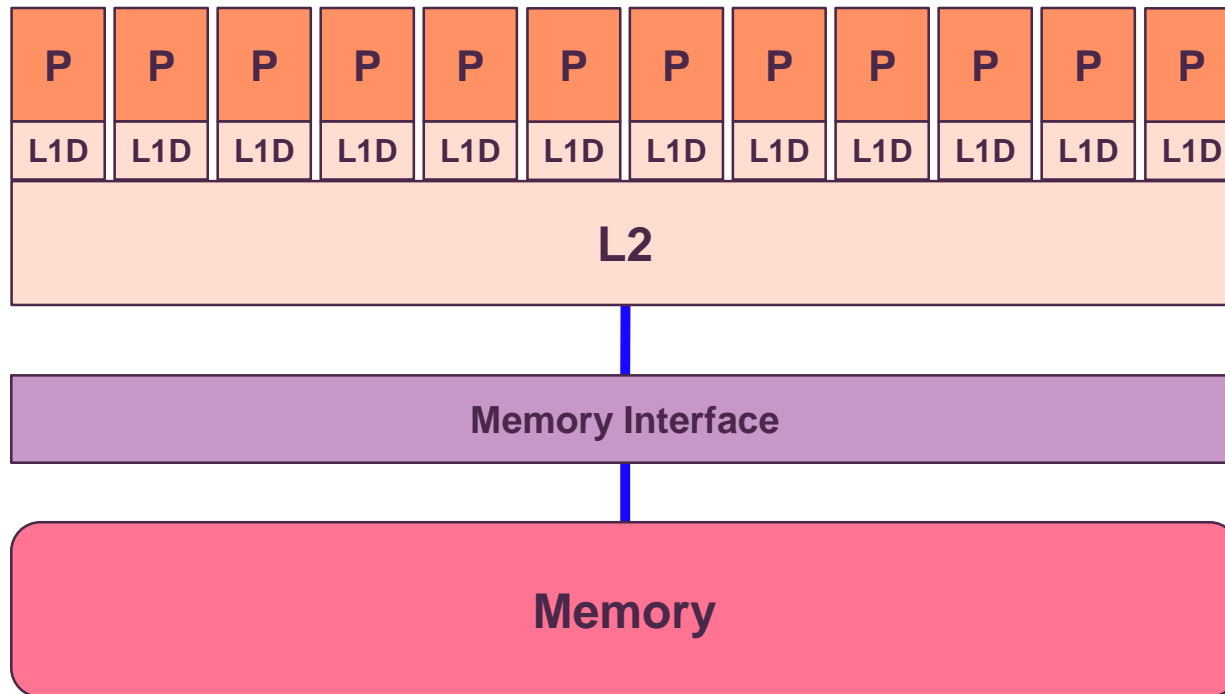


The new top system, Fugaku, turned in a High Performance Linpack (HPL) result of 415.5 petaflops, besting the now second-place Summit system by a factor of 2.8x. Fugaku, is powered by Fujitsu's 48-core A64FX SoC, becoming the first number one system on the list to be powered by ARM processors. In single or further reduced precision, which are often used in machine learning and AI applications, Fugaku's peak performance is over 1,000 petaflops (1 exaflops). The new system is installed at RIKEN Center for Computational Science (R-CCS) in Kobe, Japan.

A64FX – FX700

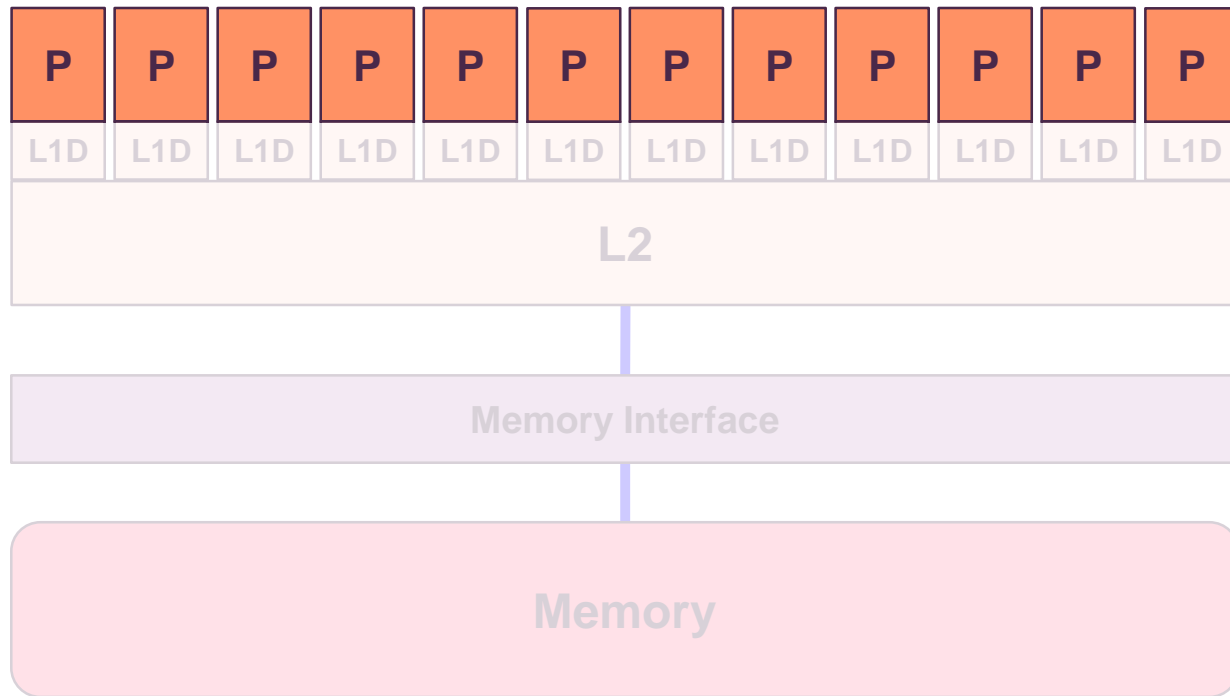


A64FX – FX700



1 CMG

A64FX – FX700



1 CMG

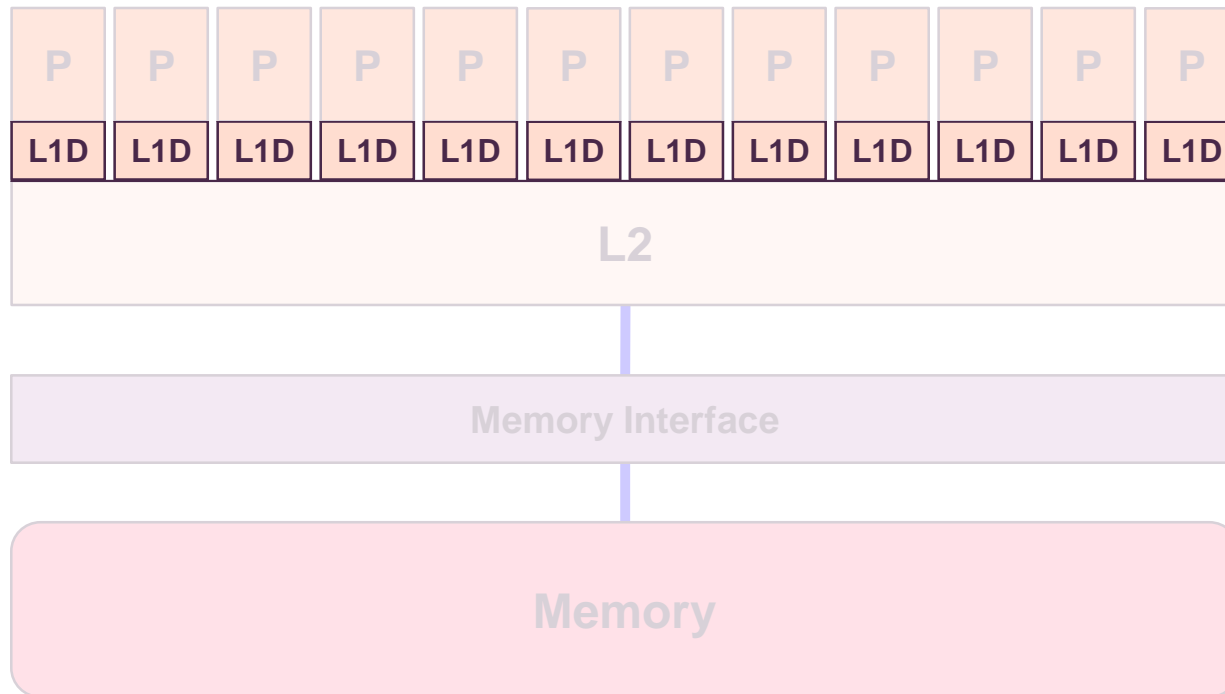
Core

- Clock : 1.8 GHz
- Instruction set : Armv8.2-A+SVE
- Maximum VL : 512 bit (8 double)

For example on GCC compiler use :

`-msve-vector-bits=512 -march=armv8.2-a+sve`

A64FX – FX700

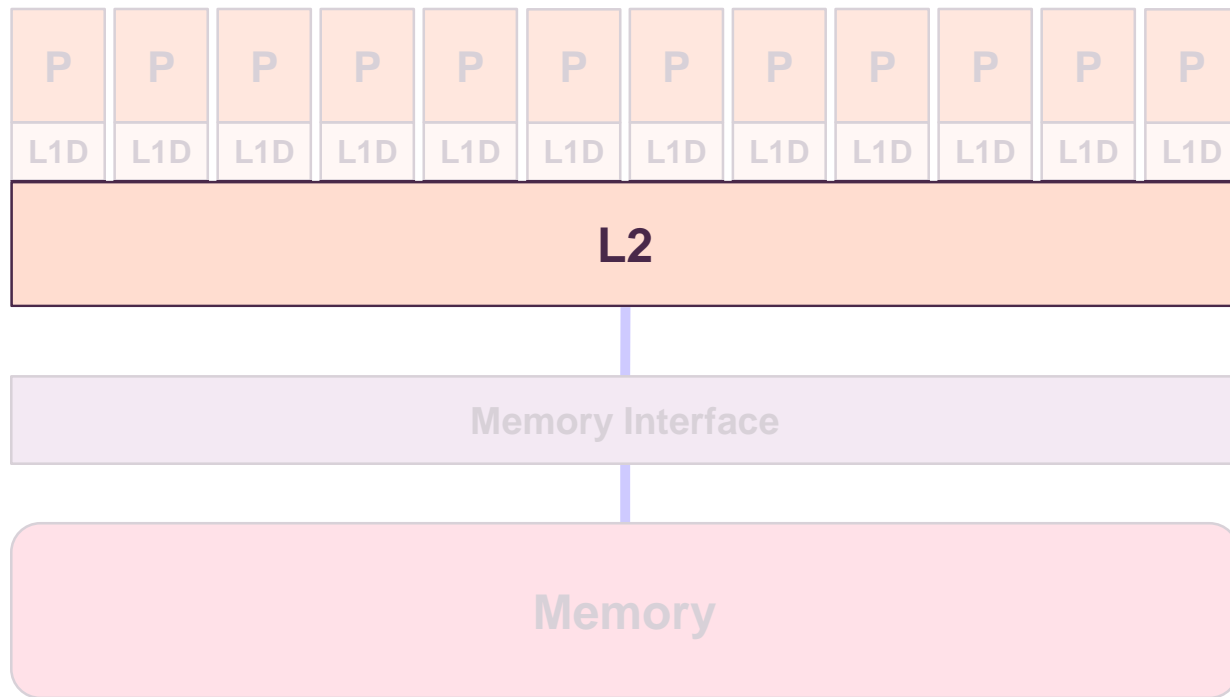


1 CMG

L1D cache

- Size : 64 KiB
- Topology : Private cache
- Cache line size : 256 bytes

A64FX – FX700

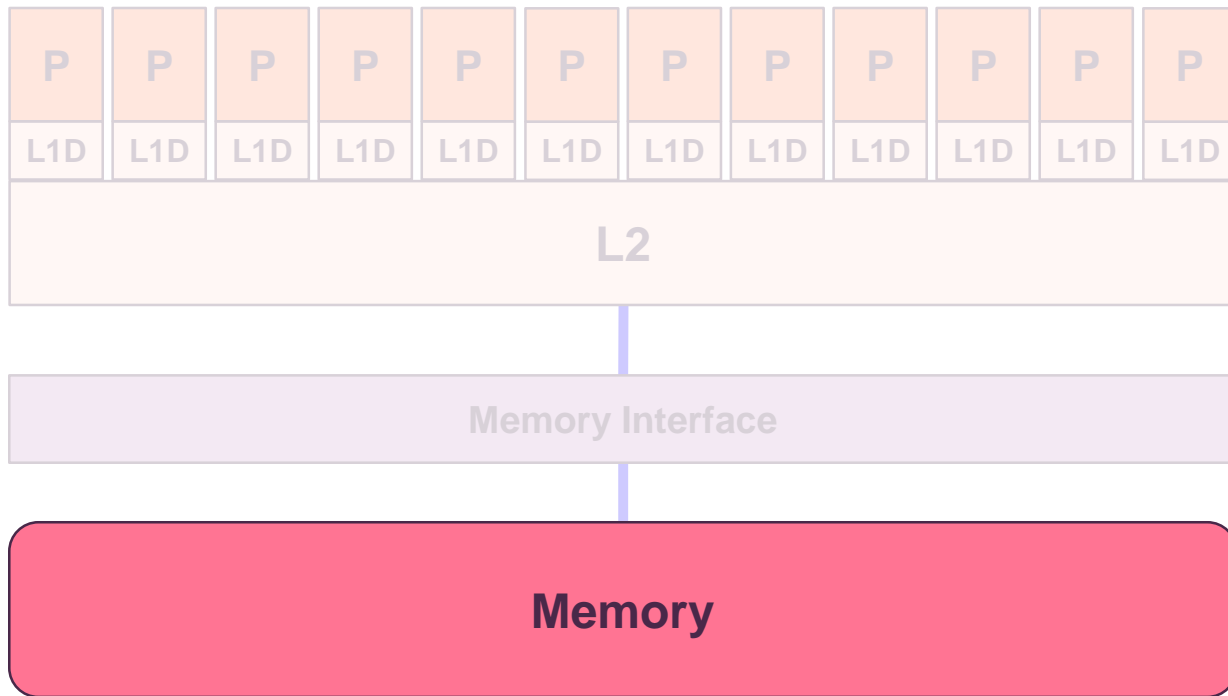


L2 cache

- Size : 8 MiB
- Topology : Shared within 1 CMG
- Cache line size : 256 bytes

1 CMG

A64FX – FX700



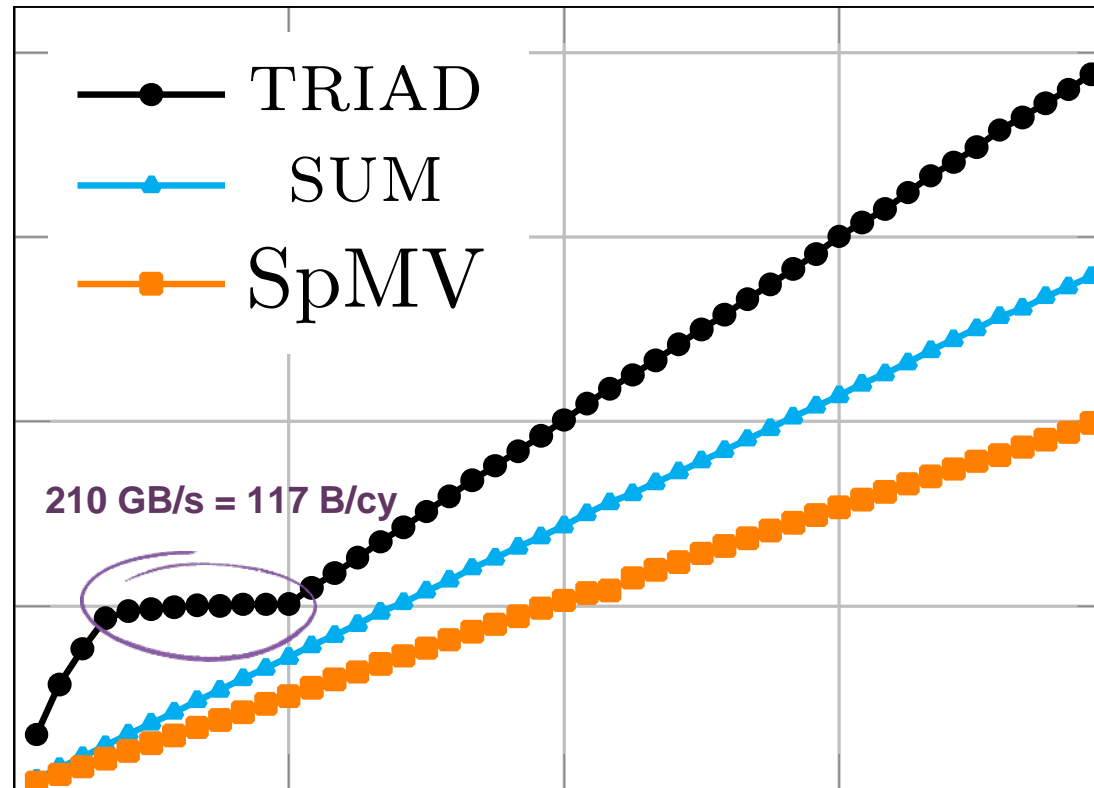
1 CMG

Main memory

- Size : 4 x 8 GiB
- Type : HBM2

Motivation

dwidth [Gbyte/s]

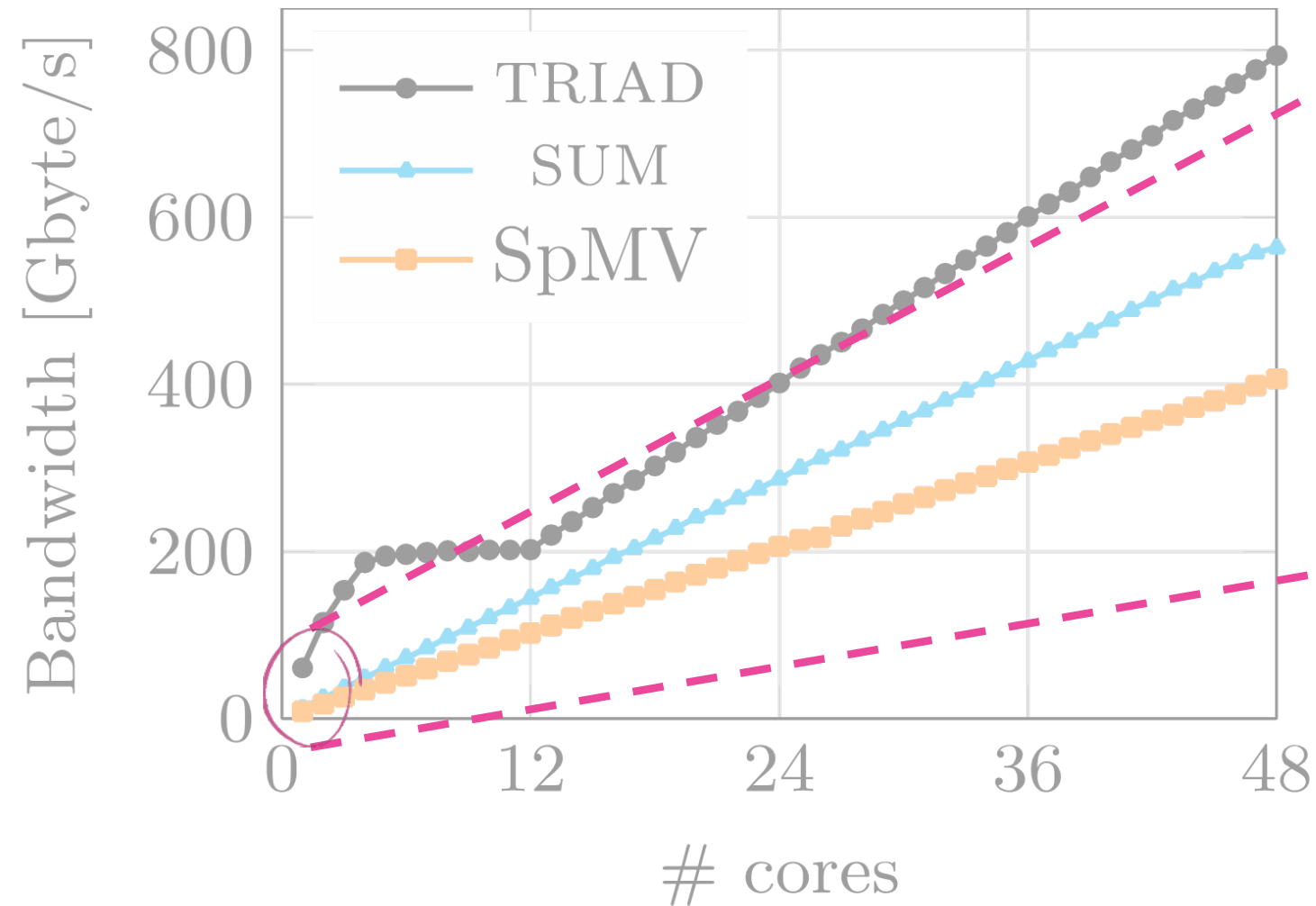


Clear memory bandwidth
saturation for STREAM TRIAD
($a[i] = b[i] + s * c[i]$).

But why not for SUM ($s += a[i]$)
and SpMV ($b = Ax$)?

Thread pinning : Compact

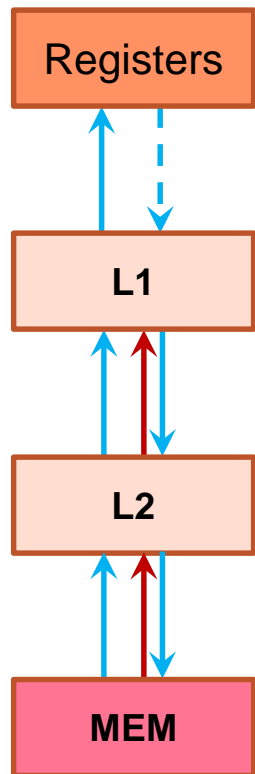
Motivation



Understanding single-core performance is the key !

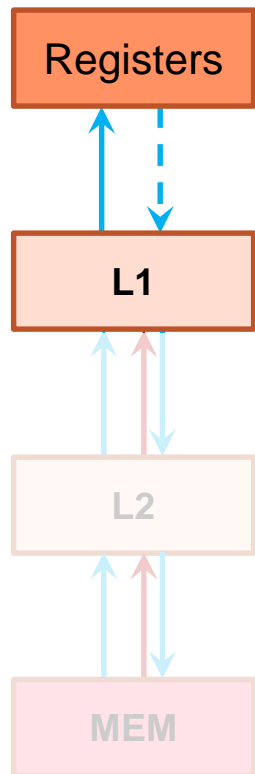
Thread pinning : Compact

Motivation → ECM model



Execution-Cache-Memory (ECM) model helps us to understand and analyze the single-core performance.

Motivation → ECM model

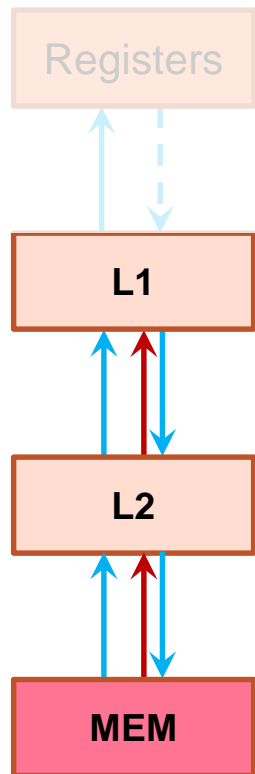


Execution-Cache-Memory (ECM) model helps us to understand and analyze the single-core performance.

3 major components :

1) In-core

Motivation → ECM model

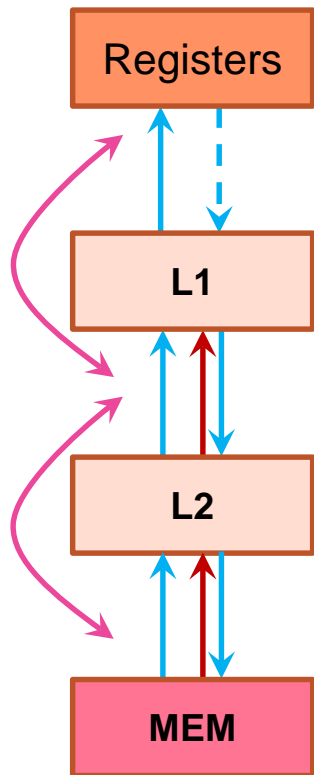


Execution-Cache-Memory (ECM) model helps us to understand and analyze the single-core performance.

3 major components :

- 1) In-core
- 2) Data transfer through memory hierarchies

Motivation → ECM model



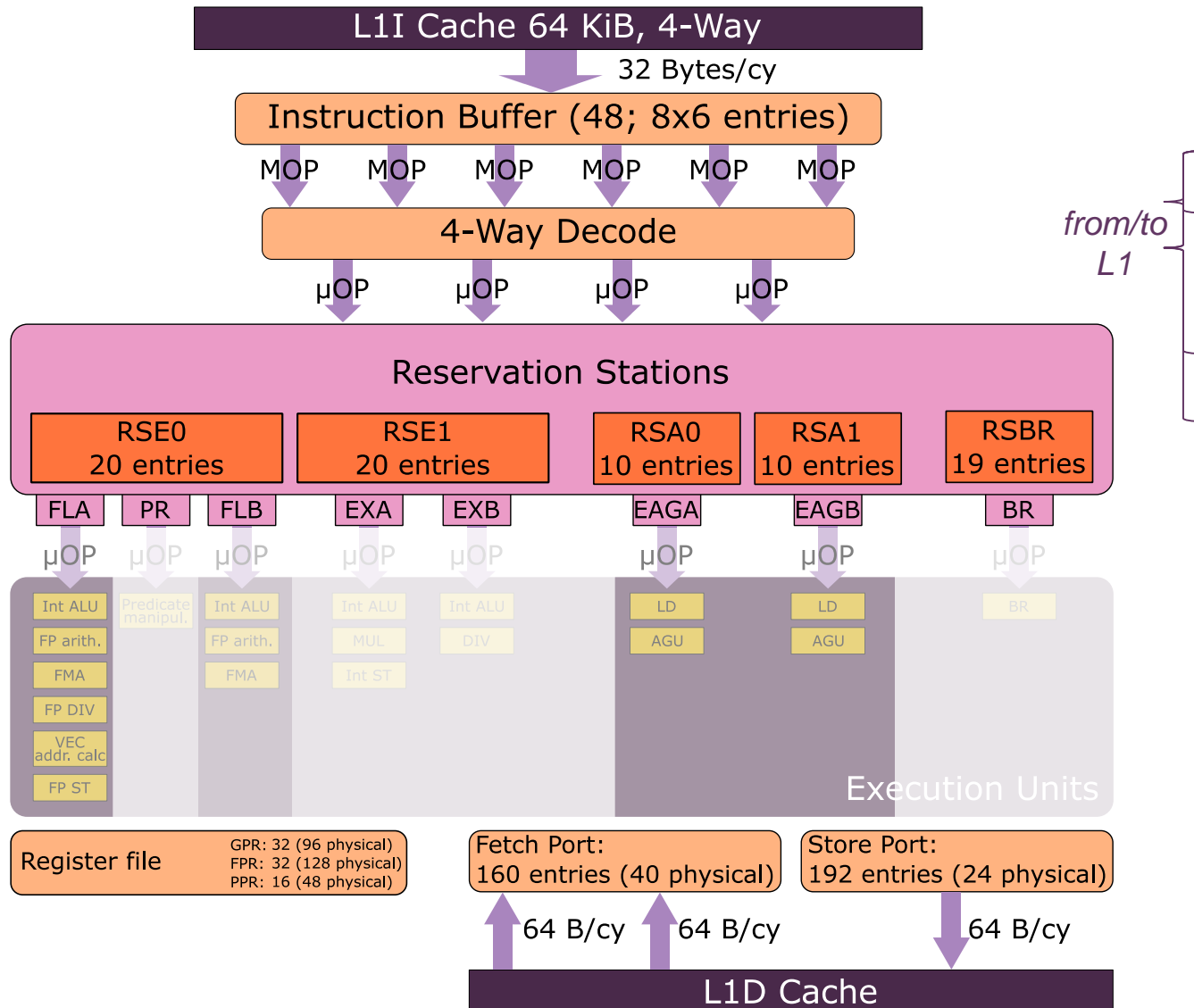
Execution-Cache-Memory (ECM) model helps us to understand and analyze the single-core performance.

3 major components :

- 1) In-core
- 2) Data transfer through memory hierarchies
- 3) Overlap hypothesis

Can these transfers be overlapped or not ?

ECM model → In-core



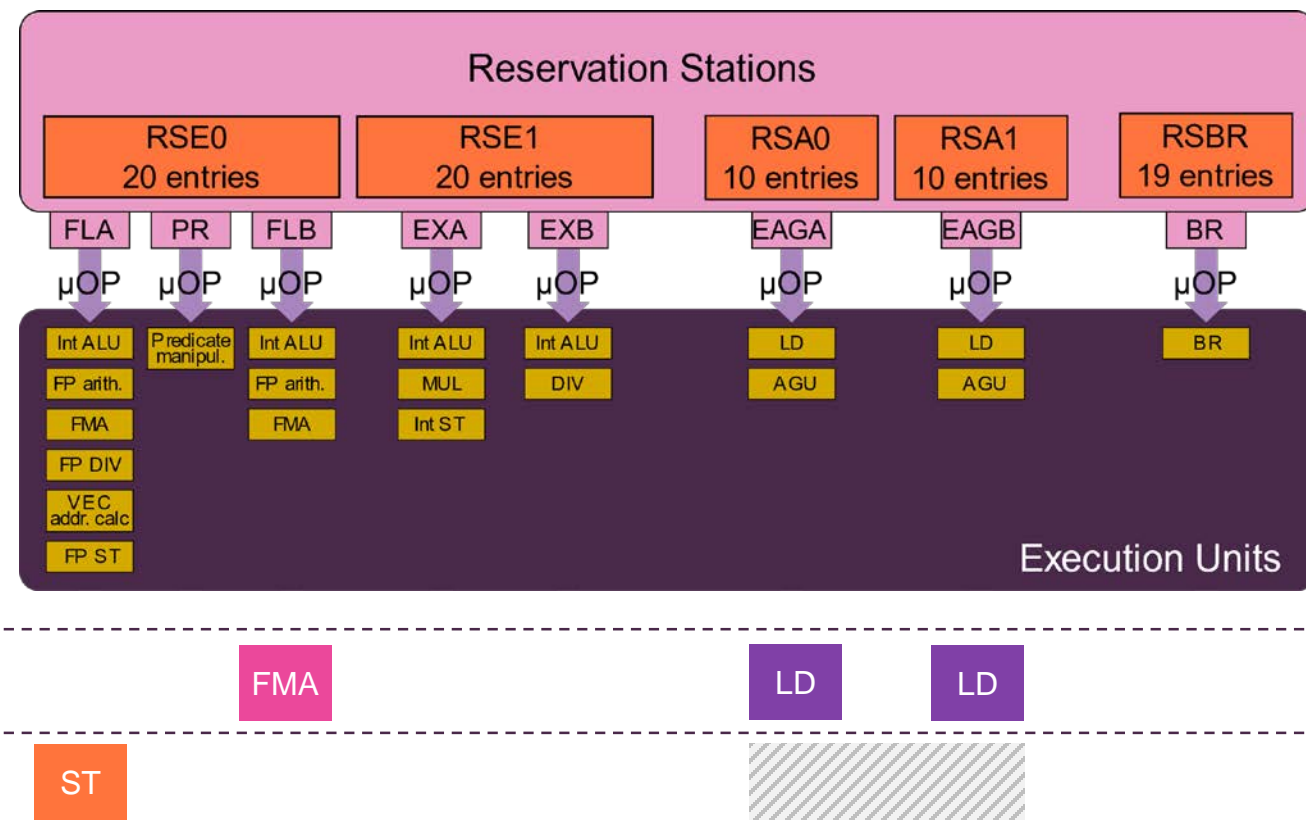
STREAM TRIAD

$$a[i] = b[i] + s * c[i]$$

.L18:

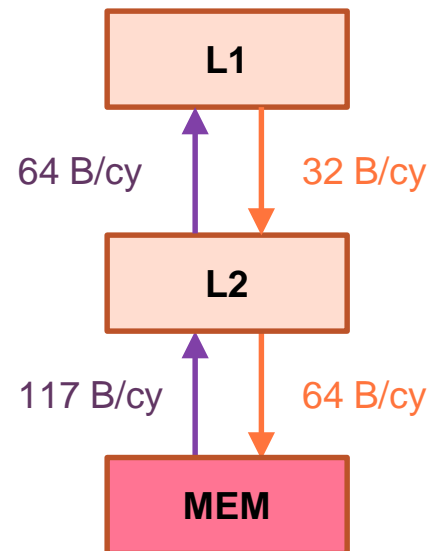
```
ld1d z4.d, p5/z, [x21, x9, lsl 3]
ld1d z5.d, p5/z, [x20, x9, lsl 3]
fmad z5.d, p5/m, z2.d, z4.d
st1d z5.d, p5, [x19, x9, lsl 3]
add x8, x9, 8
whilelo p5.d, w8, w7
b.any .L18
```

2cy / VL

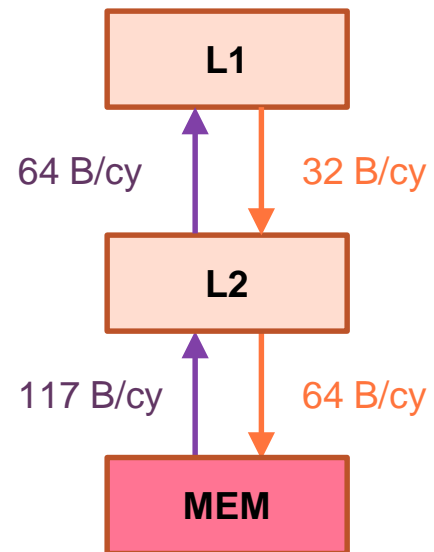


ECM model → Memory hierarchy

Machine model
FX700

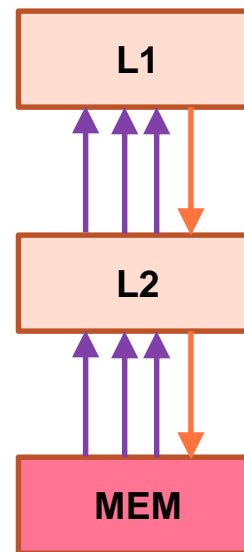


ECM model → Memory hierarchy

Machine model
FX700

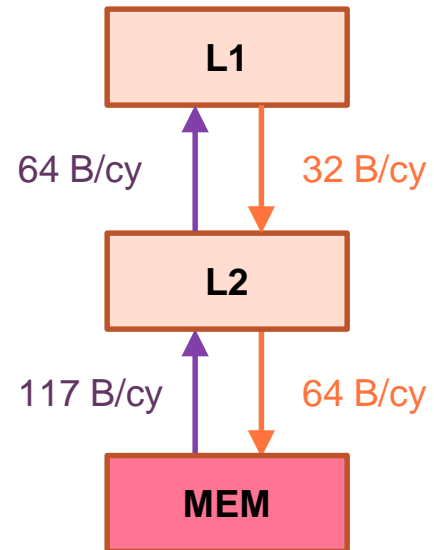
Application model

$$\text{TRIAD}$$
$$a[i] = b[i] + s * c[i]$$

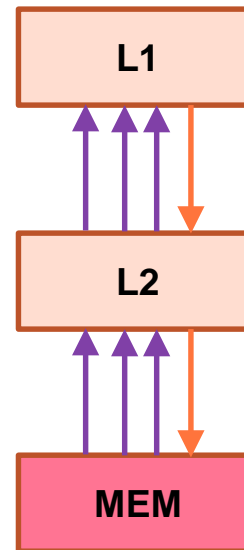


ECM model → Memory hierarchy

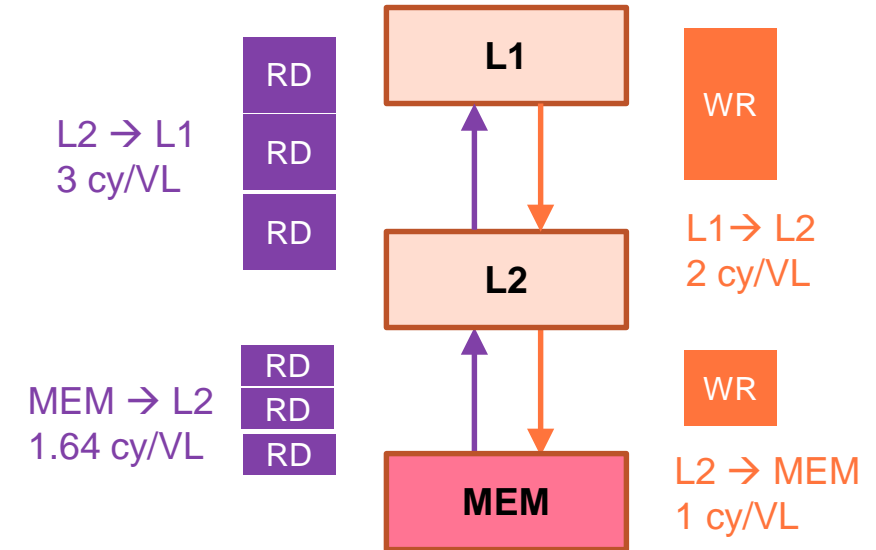
Machine model FX700



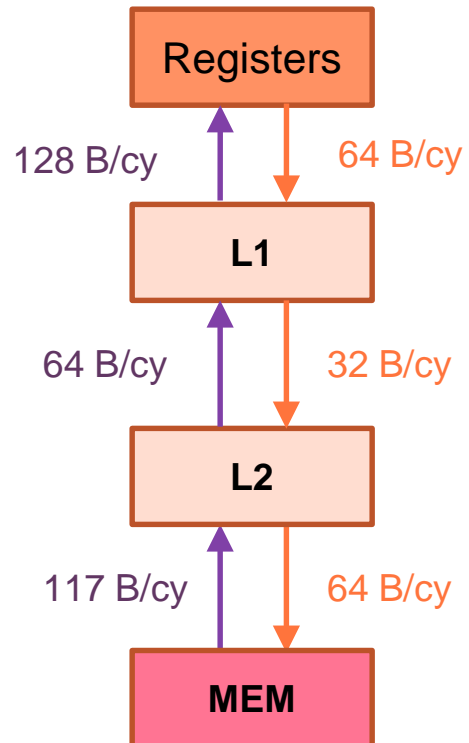
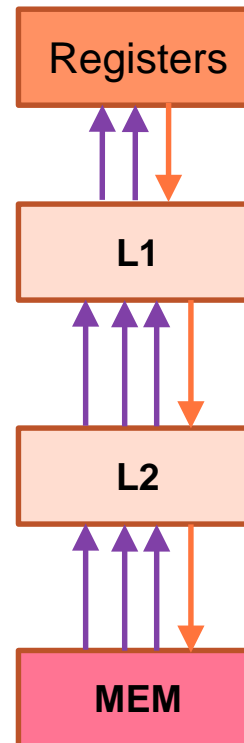
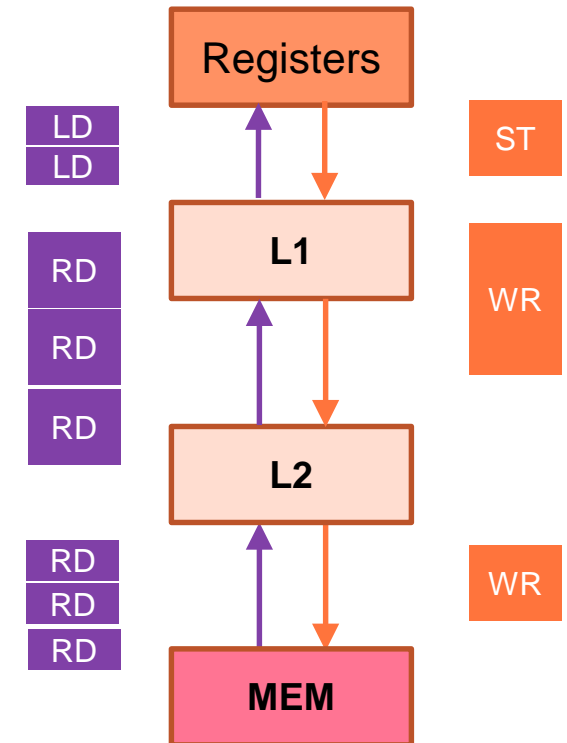
Application model TRIAD $a[i] = b[i] + s*c[i]$



ECM prediction TRIAD on FX700



ECM model → Memory hierarchy + In-core

Machine model
FX700Application model
TRIAD
 $a[i] = b[i] + s*c[i]$ ECM prediction
TRIAD on FX700

ECM model → Overlap hypothesis

How do these
boxes overlap?

ECM prediction
TRIAD on FX700

LD
LD

ST

RD

WR

RD

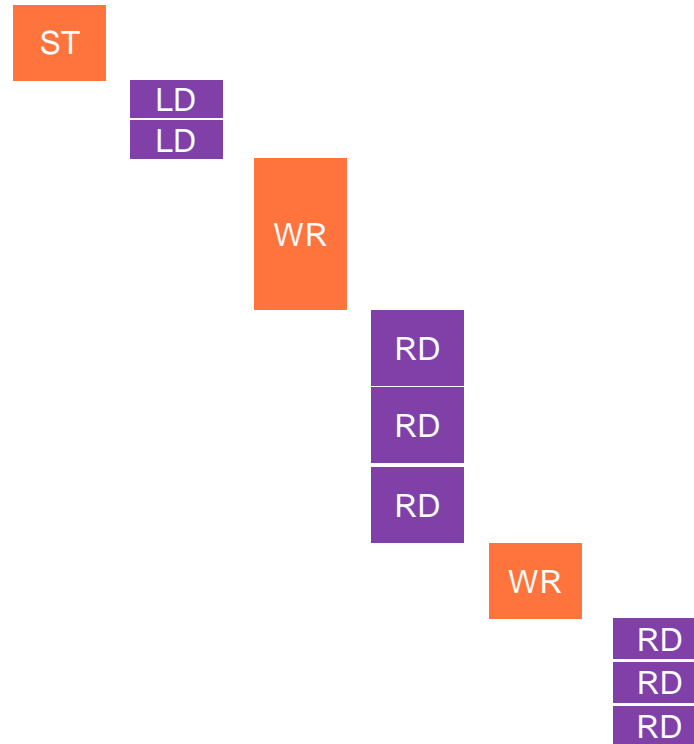
RD

RD
RD
RD

WR

ECM model → Overlap hypothesis

Hypothesis 1 : No overlap



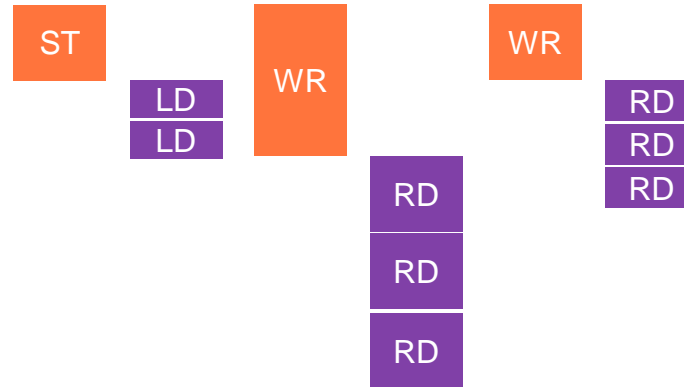
ECM model → Overlap hypothesis

Hypothesis 2 : Full overlap



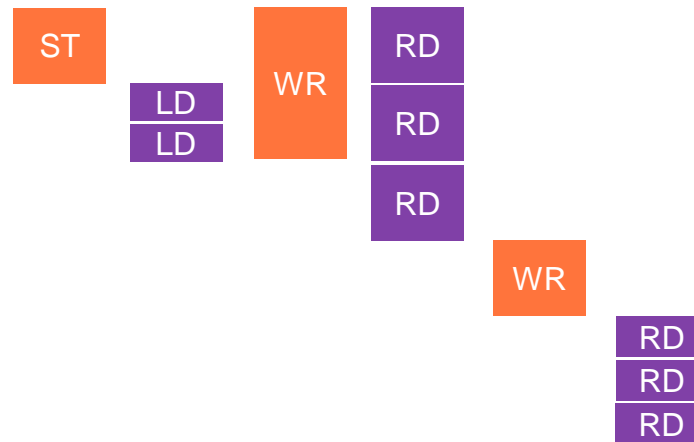
ECM model → Overlap hypothesis

Hypothesis 3 : Full overlap + half-duplex



ECM model → Overlap hypothesis

Hypothesis 4 : L1L2 overlap + half-duplex at MEM



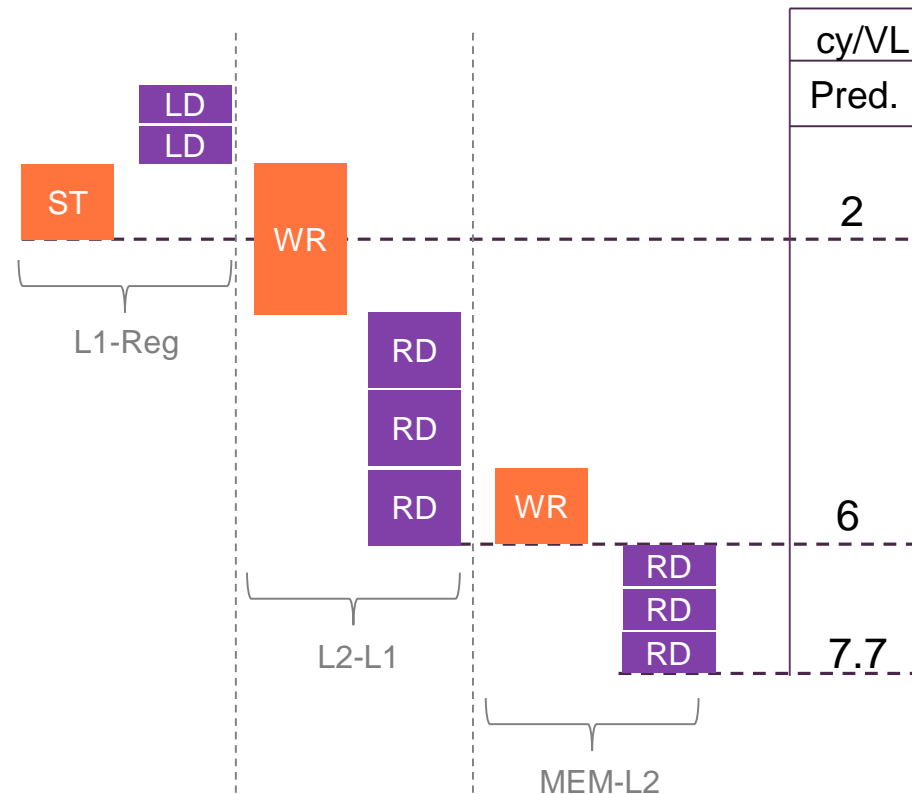
There are numerous combinations.

How do we find the correct one?

Compare measurements
with predictions.

ECM model → Overlap hypothesis

The best hypothesis for FX700



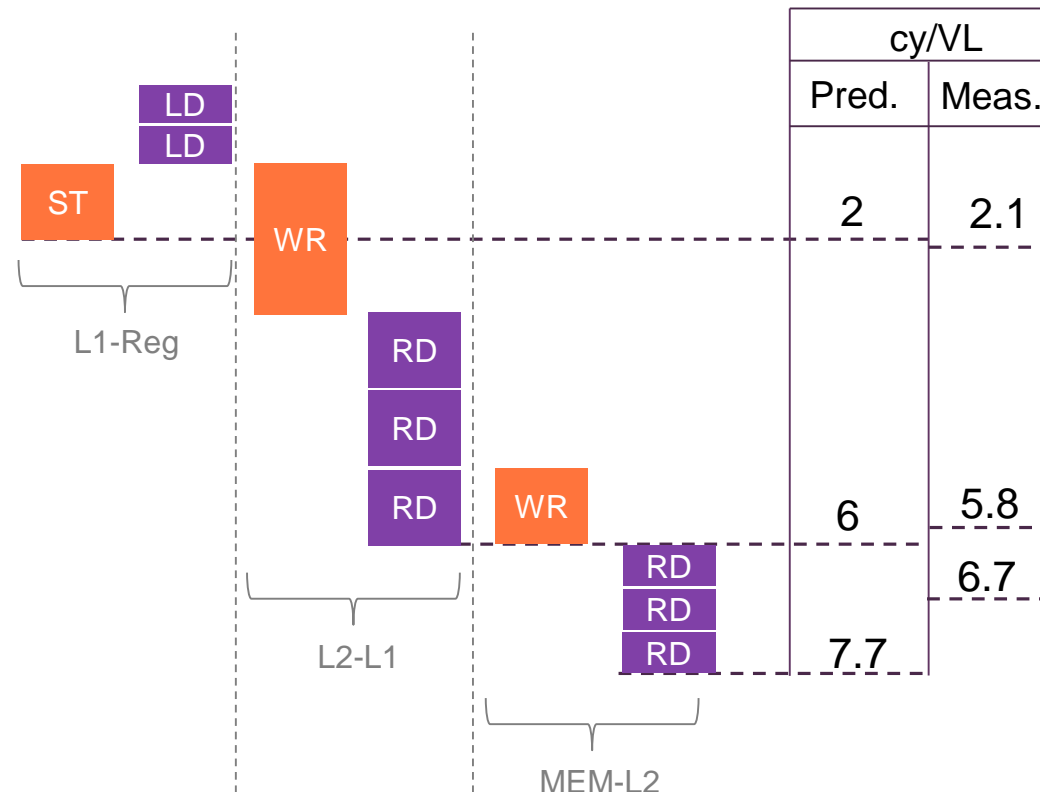
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ECM model → Overlap hypothesis

The best hypothesis for FX700



There are numerous combinations.

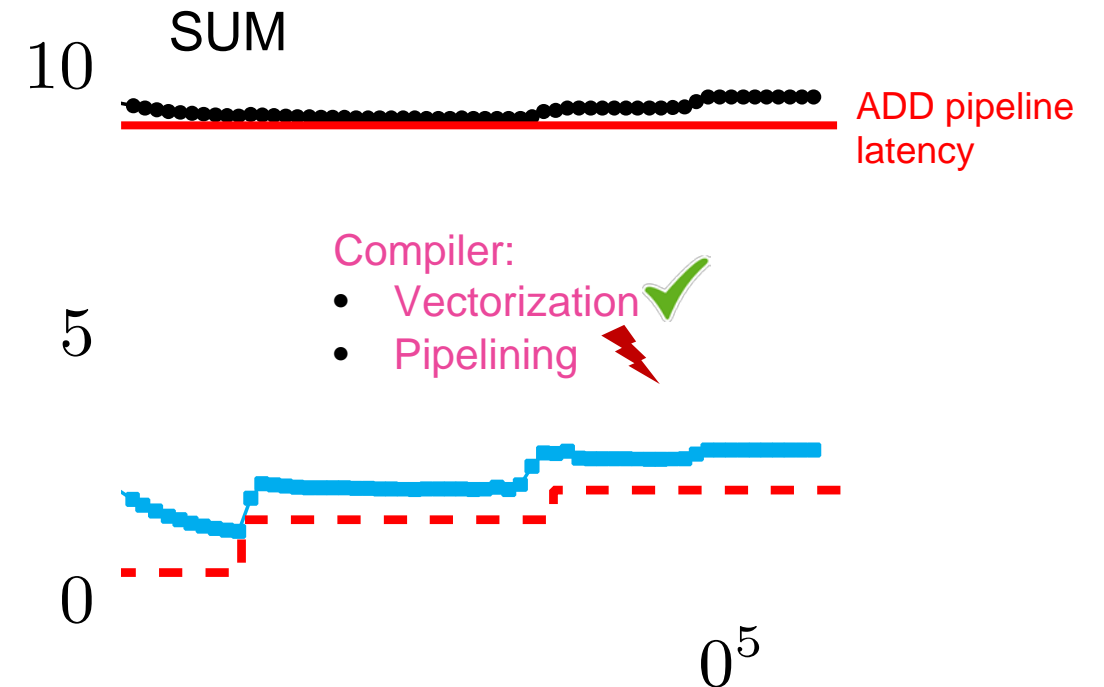
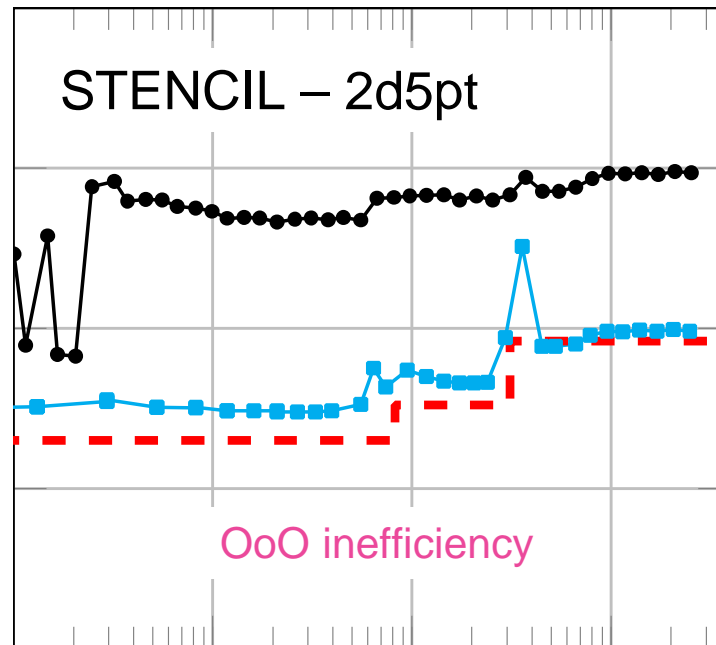
How do we find the correct one?

Compare measurements
with predictions.

A systematic way of identifying overlap hypothesis is presented in : Hofmann et.al., 2020, Bridging The Architecture Gap: Abstracting Performance-relevant Properties Of Modern Server Processors, <https://doi.org/10.14529/jsfi200204>

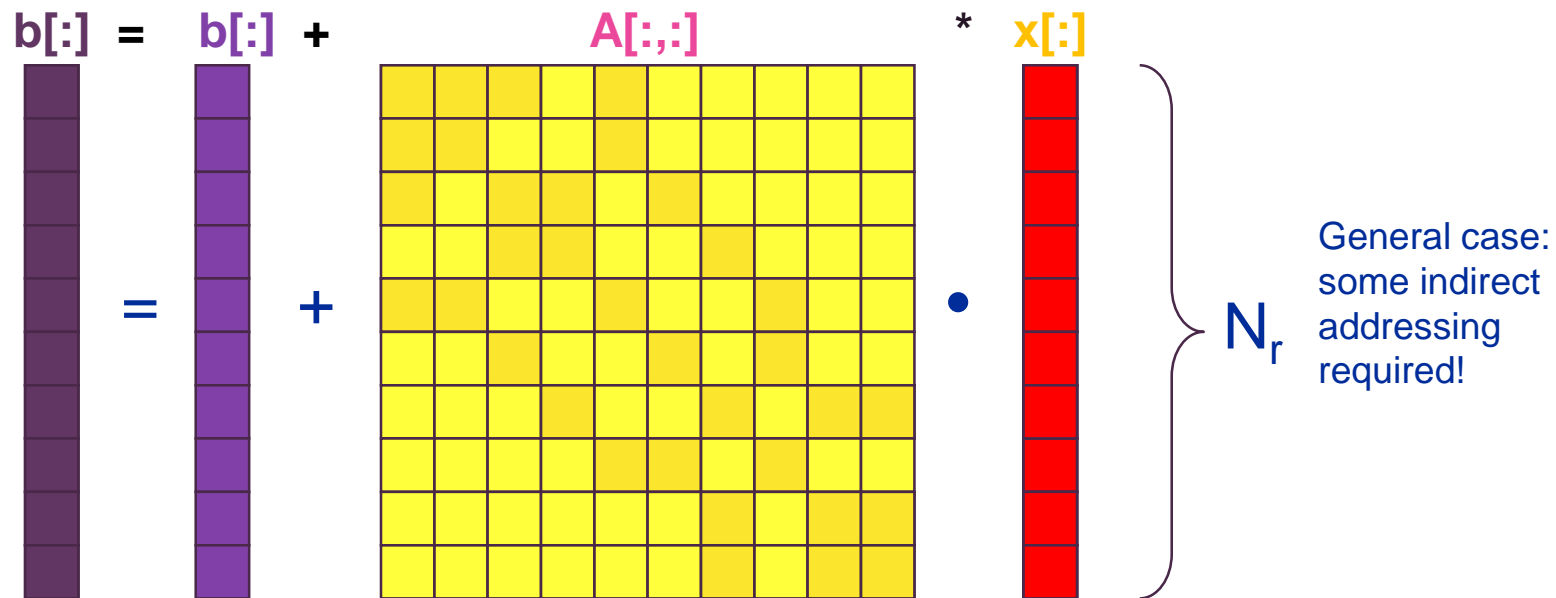
ECM model → Insights

Unrolling plays an important role



■ Unrolling factor=1 ■ Unrolling factor=8 ■ ECM prediction

Sparse Matrix-Vector Multiplication (SpMV) : $b = Ax$



In Compressed Row Storage (CRS) format

```
for i = 0:nrows-1 //Long outer-loop
    for j = row_ptr[i]:row_ptr[i+1]-1 // Short inner-loop
        b[i] = b[i] + A[j] * x[col_idx[j]]
```

Assembly of the short inner-loop

```
.L6:  
    ld1sw    z0.d, p0/z, [x17, x20, lsl 2]  
    ld1d     z2.d, p0/z, [x18, x20, lsl 3]  
    ld1d     z3.d, p0/z, [x30, z0.d, lsl 3]  
    add      x20, x20, 8  
    fmla     z1.d, p0/m, z3.d, z2.d  
    whilelo  p0.d, x20, x14  
    b.any    .L6  
  
    faddv    d4, p1, z1.d
```

GCC compiler



In Compressed Row Storage (CRS) format

```
for i = 0:nrows-1 //Long outer-loop  
    for j = row_ptr[i]:row_ptr[i+1]-1 // Short inner-loop  
        b[i] = b[i] + A[j] * x[col_idx[j]]
```

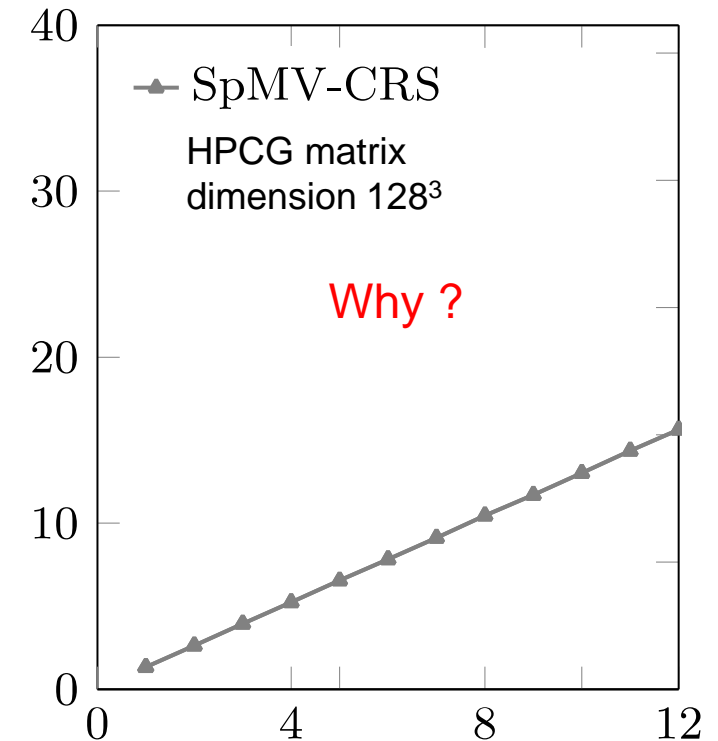
Assembly of the short inner-loop

```

.L6:
    ld1sw    z0.d, p0/z, [x17, x20, lsl 2]
    ld1d     z2.d, p0/z, [x18, x20, lsl 3]
    ld1d     z3.d, p0/z, [x30, z0.d, lsl 3]
    add      x20, x20, 8
    fmla     z1.d, p0/m, z3.d, z2.d
    whilelo  p0.d, x20, x14
    b.any    .L6

    faddv    d4, p1, z1.d

```



In Compressed Row Storage (CRS) format

```

for i = 0:nrows-1 //Long outer-loop
    for j = row_ptr[i]:row_ptr[i+1]-1 // Short inner-loop
        b[i] = b[i] + A[j] * x[col_idx[j]]
    end
end

```


Assembly of the short inner-loop

.L6:

```

ld1sw    z0.d, p0/z, [x17, x20, lsl 2]
ld1d     z2.d, p0/z, [x18, x20, lsl 3]
ld1d     z3.d, p0/z, [x30, z0.d, lsl 3]
add       x20, x20, 8
fmla     z1.d, p0/m, z3.d, z2.d
whilelo   p0.d, x20, x14
b.any     .L6

```

```

faddv     d4, p1, z1.d

```

FMA3: Update **z1.d**

Latency: 9 cycles

Loop length : 27
HPCG matrixHorizontal add of 512-bit
register

Throughput = 11.5 cycles

In Compressed Row Storage (CRS) format

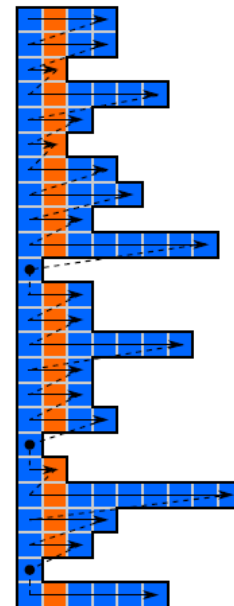
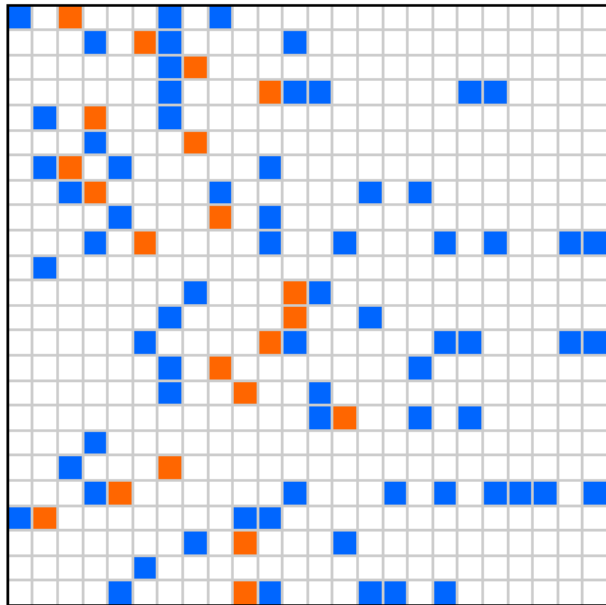
```

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        b[i] = b[i] + A[j] * x[col_idx[j]]
    end
end

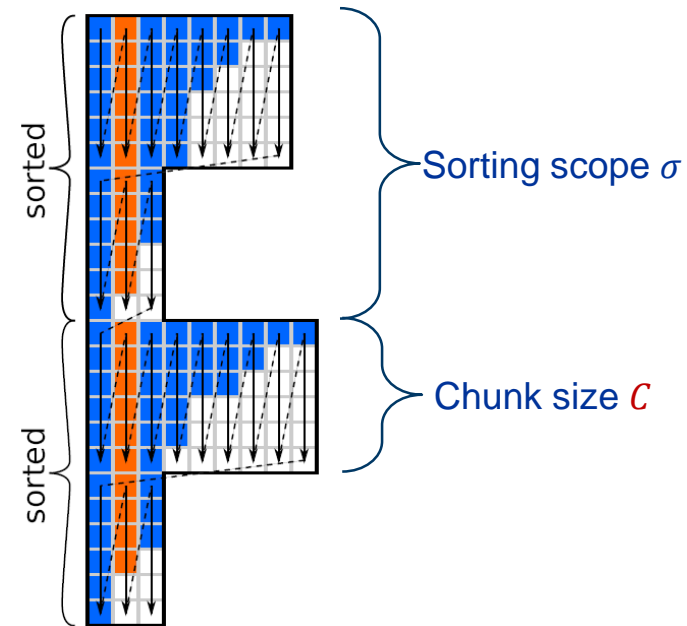
```

SpMV \rightarrow SELL-C- σ

Vector-friendly SpMV data format/kernel required for A64FX \rightarrow SELL-C- σ ¹



CRS

SELL-C- σ

Benefits:

- Vectorization and unrolling along chunk size (C) \rightarrow long loop and tunable
- No costly horizontal-add (faddv)

¹M. Kreutzer et al., A Unified Sparse Matrix Data Format For Efficient General Sparse Matrix-vector Multiplication On Modern Processors With Wide SIMD Units, SIAM SISC 2014, DOI: [10.1137/130930352](https://doi.org/10.1137/130930352)

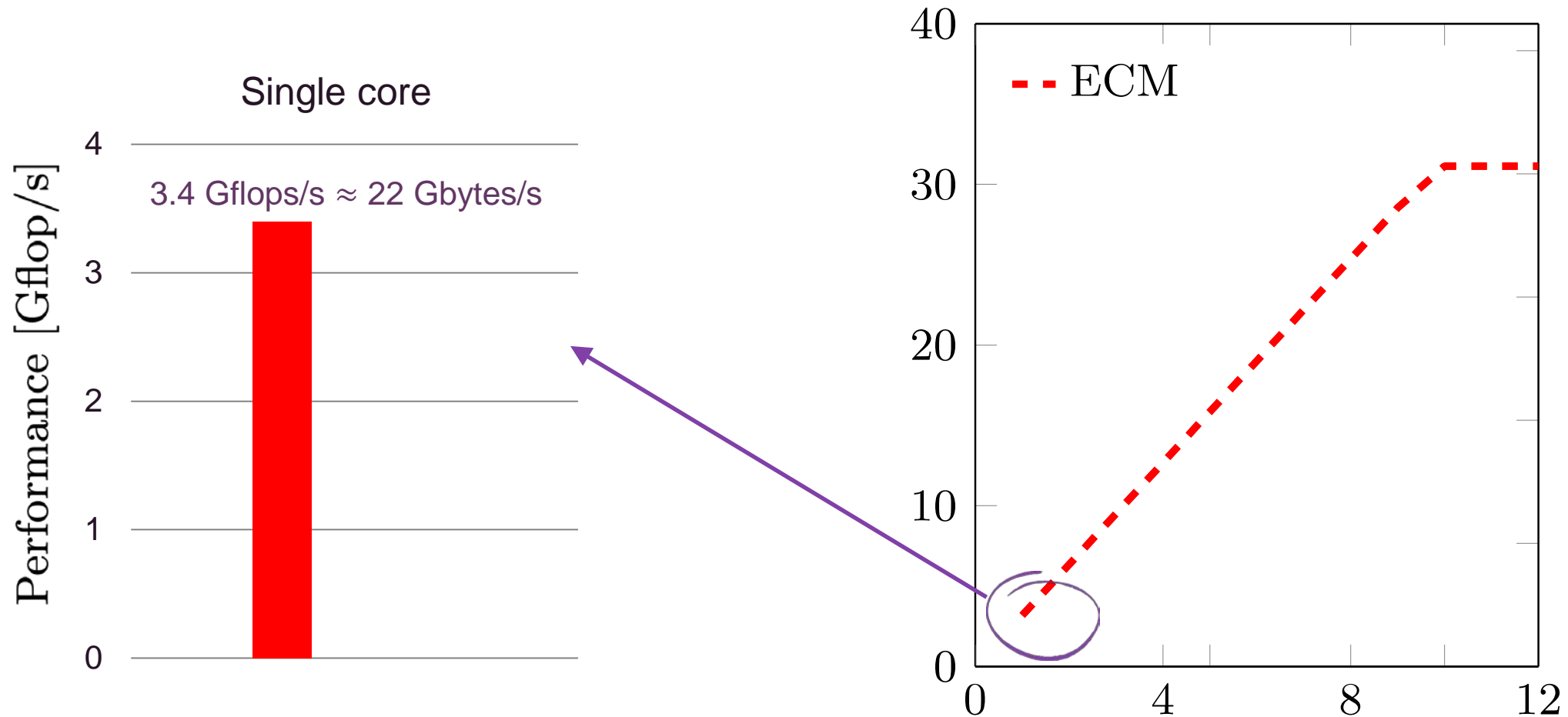
SpMV \rightarrow SELL-C- σ \rightarrow ECM



= 3.4 Gflops/s \approx 22 Gbytes/s

SpMV \rightarrow SELL-C- σ \rightarrow ECM

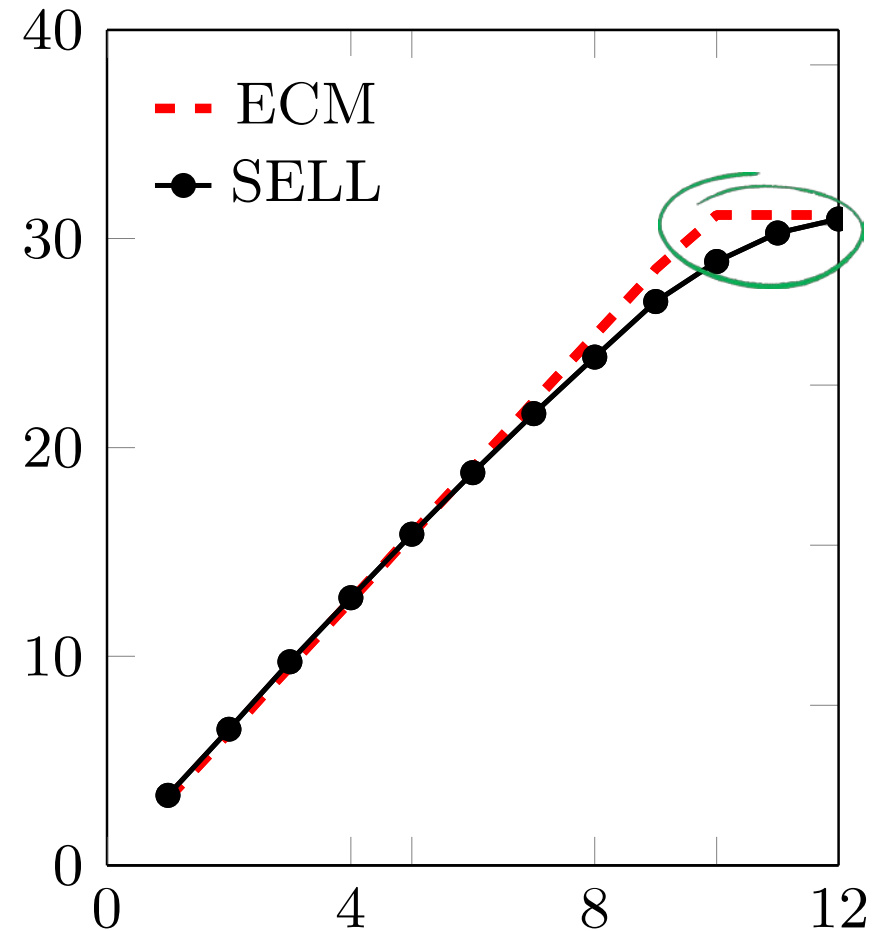
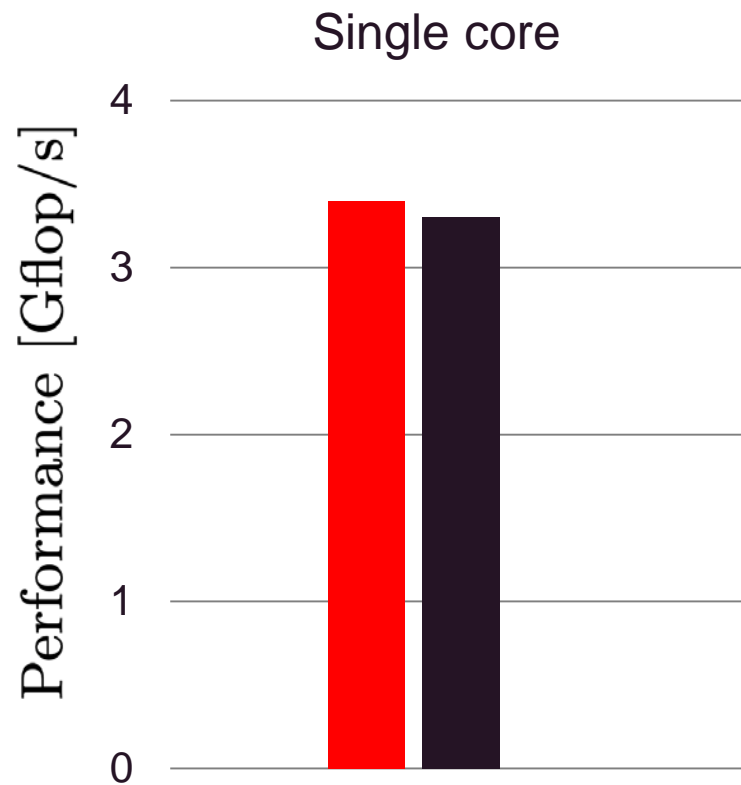
Can we saturate now ?



HPCG matrix, dimension 128^3

SpMV \rightarrow SELL-C- σ \rightarrow ECM

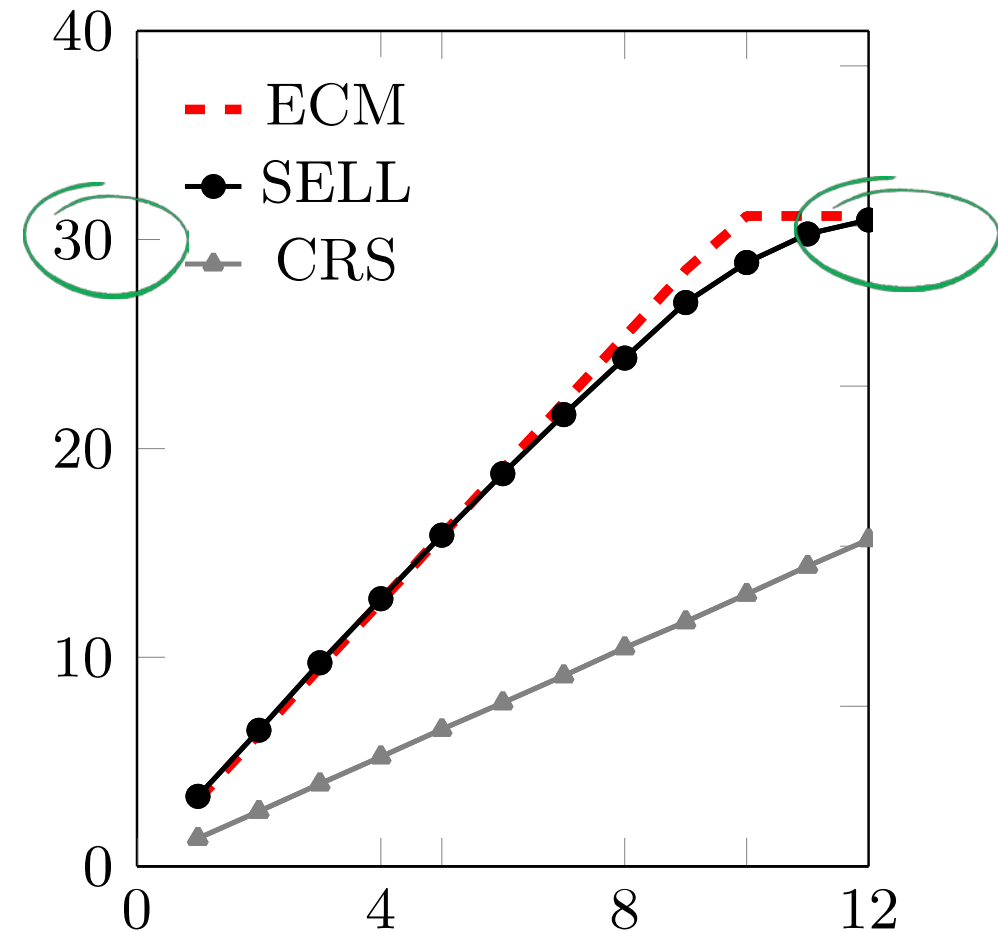
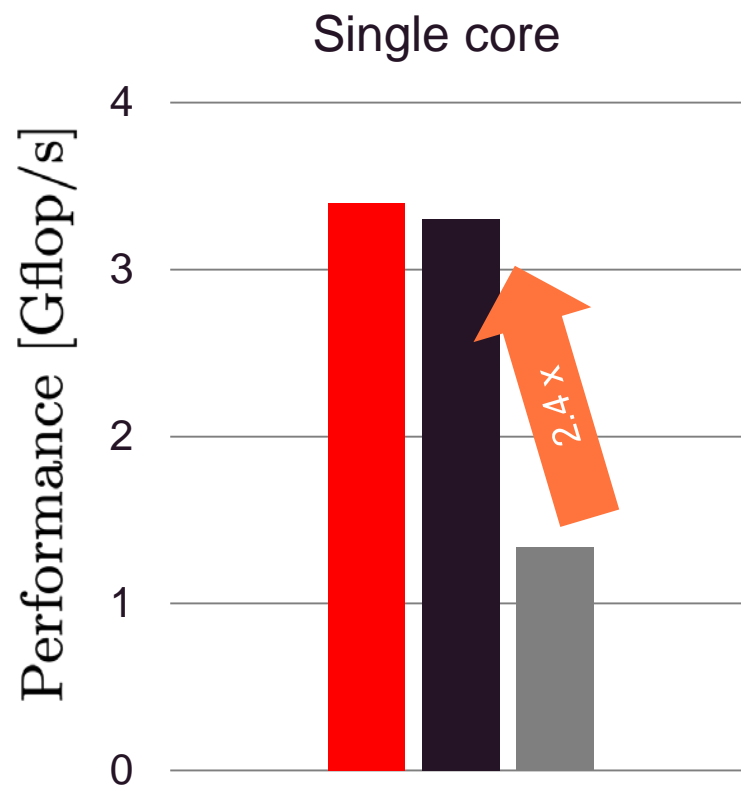
Can we saturate now ? Yes, but needs almost all cores



HPCG matrix, dimension 128^3

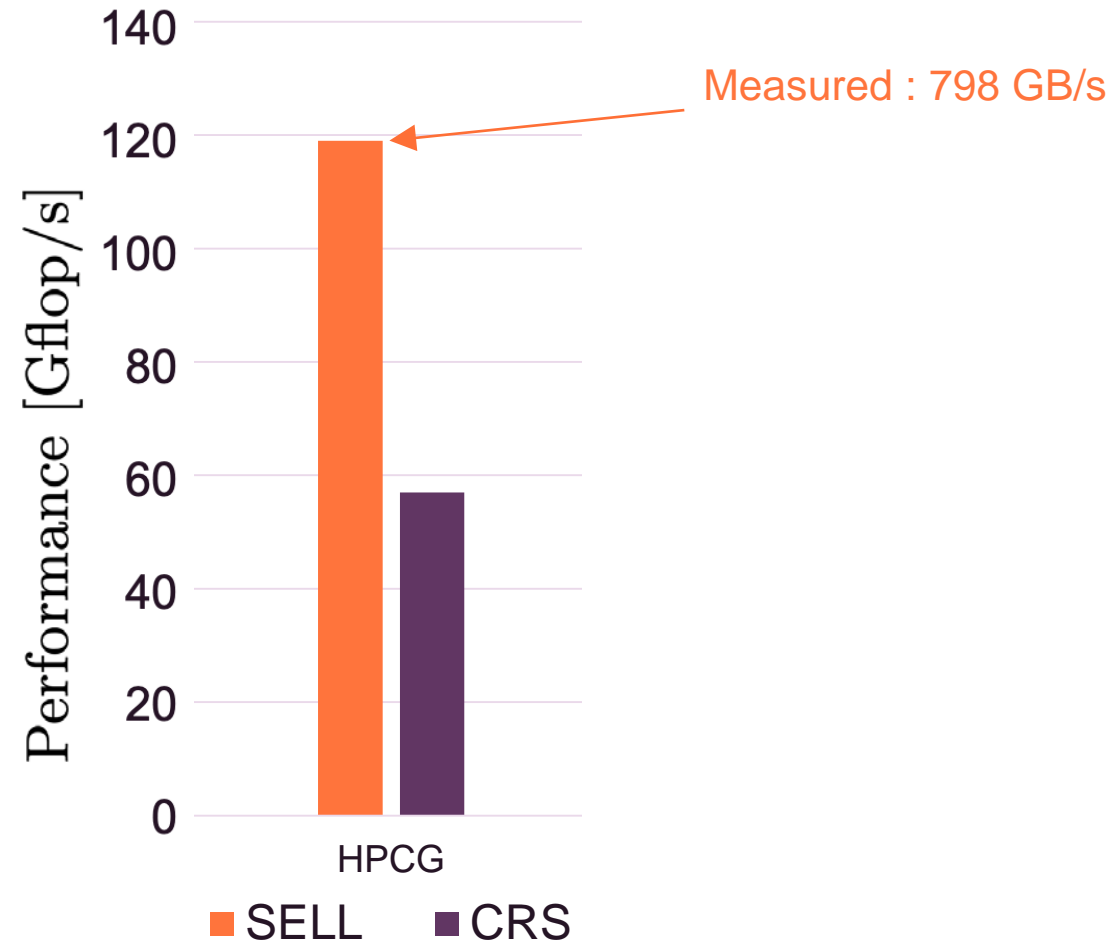
SpMV \rightarrow SELL-C- σ \rightarrow ECM

Can we saturate now ?



HPCG matrix, dimension 128^3

SpMV performance on full node (48 cores)



SpMV performance on full node (48 cores)



Conclusion

- High single core performance is crucial.
- ECM model was established and utilized to analyze the single core performance.
- The partial overlapping memory hierarchy allows for high single-core memory bandwidth.
- Proper single core optimizations have to be done to hide long floating point latency and inefficiencies in OoO.
- For SpMV we were able to saturate the bandwidth with SELL-C- σ format.

Thank you

Questions ?

KONWIHR

